Limits on the Halo White Dwarf Component of Baryonic Dark Matter from the *Hubble Deep Field*

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ABSTRACT

The MACHO collaboration lensing event statistics suggest that a significant fraction of the dark galactic halo can be comprised of baryonic matter in the form of white dwarf stars with masses between 0.1 and 1.0 $M_\odot$. Such a halo white dwarf population, in order to have escaped detection by those who observe the white dwarf luminosity function of the disk, must have formed from an old population. The observations indicate that the number of halo white dwarfs per cubic parsec per unit bolometric magnitude is less than 10^{-5} at $10^{-4.5}L_\odot$; the number must rise significantly at lower luminosities to provide the needed baryonic halo mass. Such white dwarfs may easily escape detection in most current and earlier surveys. Though it is limited in angular extent, the *Hubble Deep Field* (HDF) probes a sufficient volume of the galactic halo to provide interesting limits on the number of halo white dwarf stars, and on the fraction of the halo mass that they can make up. If the HDF field can be probed for stars down to $V = 29.8$ then the MACHO result suggests that there could be up to 12 faint halo white dwarfs visible in the HDF. Finding (or not finding) these stars in turn places interesting constraints on star formation immediately following the formation of the galaxy.

Subject headings: cosmology: dark matter — stars: white dwarfs

1. Introduction

Two recent investigations that have exciting consequences over a broad spectrum of modern astrophysics were announced at the American Astronomical Society meeting in January 1996. First, the MACHO collaboration released a summary of their observations of microlensing events in the Milky Way, interpreting the profiles of 7 long-duration lensing

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events in the direction of the LMC as evidence for lensing by objects with masses between 0.1 and 1.0 $M_\odot$(Bennett et al. 1995, 1996). The lensing objects must be of extremely low luminosity; therefore ordinary dwarf stars are ruled out. They suggest that about 50% of the dark matter halo of the Milky Way could therefore be comprised of white dwarf stars. Considering the large mass of the galactic halo, this implies that halo white dwarfs must be extremely abundant. Direct observational constraints on the halo white dwarf luminosity function from observations (Liebert et al. 1988) are not necessarily in conflict with the MACHO result; however the luminosity function of white dwarfs in the disk must then contain a fair fraction of halo white dwarfs.

Given the age of the halo as determined from, for example, globular cluster studies, the observed downturn in the white dwarf luminosity function at $\log(L/L_\odot) \approx -4.5$ ($M_{\rm bol} \approx 16.0$, with no white dwarfs with $M_{\rm bol} > 16.2$) constrains the star formation history for the generation of halo stars that might have produced such a halo white dwarf population (Tamanaha et al 1990, Adams & Laughlin 1996). Under conventional assumptions about star formation early in the history of the galaxy, Adams & Laughlin (1996) conclude that the observations of Liebert et al. (1988) already limit the fraction of the dark matter halo that can be attributed to white dwarfs to less than 25%. On the other hand, Tamanaha et al. (1990) show that extreme conditions, such as an enormous burst of star formation early in the history of the galaxy, could produce a massive number of white dwarfs in the halo that could escape detection.

The second investigation summarized at the meeting was the Hubble Deep Field (HDF) (Williams et al. 1995). The HDF is an extremely deep set of images obtained with the Hubble Space Telescope in a relatively blank field out of the galactic plane. The HDF has quoted 3σ detection limits of approximately 30th magnitude in the V and I plates. Though a very narrow angle survey, the depth of the HDF results in its sampling a significant volume of the halo. Thus it is useful for the purposes of detecting (or placing upper limits on the distribution of) halo white dwarf stars. This Letter discusses the use of the HDF to constrain the luminosity function of halo white dwarfs. It outlines the computation of the number of white dwarfs expected in the HDF as a function of the limiting magnitude for detection of stars and the luminosity of the faintest expected white dwarfs. As discussed in the final section, identification of stellar candidates to near the limit of sensitivity of the HDF will provide significant constraints on the fraction of the halo mass that can be attributed to white dwarf stars, or on the luminosity function of such stars.

2. The Halo as Probed by the HDF

For a small survey area such as the HDF, the volume V of space sampled out to a distance d (in pc) given an area of A square arc minutes is simply

$$V = 2.82 \times 10^{-8} A d^3 \ \text{pc}^3 \ . \tag{1}$$

The HDF covered an area of approximately 4 square arc minutes; therefore since it looked out of the plane, taking $d = 500$ pc suggests that it samples a disk volume of only 14 pc^3. With such a small volume sample, it is not surprising that no white dwarfs are in the field; the approximate space density of white dwarf stars is 3×10^{-3} per cubic parsec (Liebert et al. 1988).

For a putative population of white dwarfs in the halo, the HDF has sampled a much larger volume. Assuming a limiting magnitude of m_l, and a white dwarf with an absolute magnitude M_f (in the same band as the limiting magnitude), then the distance out to which that white dwarf would be visible, d_f, is simply

$$\log(d_f) = 0.2(m_l - M_f + 5) \ . \tag{2}$$

With this value for the range of the survey, a lower limit for the effective volume contained within can be written in terms of the limiting magnitude of the survey and the absolute magnitude of the faintest white dwarf:

$$\log(V) = -4.550 + \log(A) + 0.6(m_l - M_f) \tag{3}$$

where V is in pc^3 and A is in square arc minutes.

Another way that the depth of a survey can be judged is by assuming a mass distribution for the halo, and then computing the halo mass contained within the survey volume. Thus if the density distribution is assumed to be spherically symmetric about the center of the galaxy, (i.e. $\rho = \rho_o f(r)$) then the mass (in M$_\odot$) contained within the survey area is

$$M_H = 9.4 \times 10^{-9} A \rho_o \int_0^{d_f} f(r) \delta^2 d\delta \tag{4}$$

The distance from the galactic center r can be expressed in terms of the integration variable δ, the distance from the Sun, with the following transformation:

$$r^2 = \delta^2 + D^2 - 2\delta D \cos b \cos l \tag{5}$$

where (b, l) is the direction of the survey in galactic coordinates, and D is the distance between the Sun and the galactic center.

For the halo structure function $f(r)$, we adopt the standard form used by Binney & Tremaine (1987):

$$f(r) = \frac{1}{1 + (r/a)^\gamma} \tag{6}$$

where a and γ, along with ρ_o, are parameters derived from dynamical studies of the Galaxy. For simplicity we take $\gamma = 2$ which allows analytic integration for the mass. With this form for the halo mass distribution, the mass of the halo sampled by the HDF field (in solar masses) can be written in terms of d_f after a bit of integration (and a lot of algebra) as

$$
\begin{aligned}
M_H \; = \; & 16.07 A \rho_o a^2 \\
& \times \; \left\{ d_f - \frac{B}{2} \ln \left(1 + \frac{d_f^2}{E^2} + \frac{B d_f}{E^2} \right) \right. \\
& \left. + \frac{B^2 - 2E^2}{C} \left[\arctan \left\{ \frac{2d_f + b}{C} \right\} - \arctan \left\{ \frac{B}{C} \right\} \right] \right\}
\end{aligned}
\tag{7}
$$

where

$$E^2 \; = \; a^2 + D^2, \tag{8}$$

$$B \; = \; -2D \cos b \cos l, \tag{9}$$

and

$$C = \sqrt{4E^2 - B^2}. \tag{10}$$

The values of B, C, D, E, a, and d_f are all in parsecs in the above expressions. For the Hubble Deep Field, $b \approx 54^o$ and $l \approx 127^o$. This fixes B as 6.01 kpc. The values of a and ρ_o depend on the precise model favored for the halo, but representative values are $a = 2$ kpc and $\rho_o = 0.19$ M$_\odot$/pc^3, from Bahcall & Soneira (1980). Alternate halo models which preserve the M/L ratio in the disk and the total halo mass out to $r = D$ are possible. For the representative values of the above parameters, the value of M_H rises from 1.05M$_\odot$ for d_f=1.0 kpc to 7.8M$_\odot$ for d_f=2.0 kpc; Table 1 shows M_H as a function of $m_l - M_f$. Thus the HDF in principle samples several solar masses of halo material.

3.   Halo White Dwarfs and the HDF

If, as suggested by the MACHO results, the halo dark matter of the Milky Way is up to 50% (by mass) halo white dwarfs, then up to half of the halo mass sampled in the HDF can be white dwarf stars. The mass sampled is dependent upon d_f which in turn is set by the absolute magnitudes of the white dwarfs and the limit of detectability in the images.

The expected number of white dwarfs will be a fraction x_f of the total number that are more luminous than the magnitude limits used to determine d_f. The number of white dwarfs that should be visible on the HDF can be estimated by assuming a reasonable mean mass for white dwarfs; Table 1 gives these numbers for a mean mass of $0.6 M_\odot$ in terms of x_f, assuming that 50% of the halo mass in the field is attributable to white dwarfs.

The fraction of halo white dwarfs brighter than a given $M_{\rm bol}$, x_f, contains the dependency of the results on the white dwarf luminosity function and therefore on the age of the halo, the history of star formation there, and the physics of white dwarf cooling (see, for example, Wood 1992). Here we consider a few simplified cases shows the range of possible values of x_f. In these cases, we simplify the inputs to the luminosity function by considering a single–mass population of white dwarfs (with $M = 0.6 M_\odot$) and ignore the (small, Adams & Laughlin 1996) effects of a distribution of initial masses, the pre-white dwarf lifetimes, etc. Under this approximation, the luminosity function is inversely proportional to the fading rate $d(\log L)/dt$, and in the event that $t_{\rm cool} = K L^\alpha$, x_f is approximated by

$$ x_f = \frac{K L_{\rm lim}^\alpha - t_{\rm young}}{t_{\rm old} - t_{\rm young}} \, , \tag{11} $$

where $L_{\rm lim}$ is the minimum luminosity white dwarf detectable, $t_{\rm old}$ is the age of the oldest white dwarf in the halo, and $t_{\rm young}$ is the age of the youngest white dwarf in the halo. Though crude, these simplifications provide a decent estimate of the range of x_f. Also, the bolometric correction is problematic for the coolest white dwarfs (Liebert et al. 1988, Wood 1992) and so it is assumed to be zero here, in line with the arguments presented in Liebert et al. (1988).

In the first limit, assume that white dwarfs cool via the Mestel (1952) law ($t_{\rm cool} = 5.5 \times 10^6 L^{-5/7}$ yr) normalized to give a cooling time for white dwarfs in the disk of 9×10^9 yr (Hansen & Kawaler 1994). In this limit, halo white dwarfs formed 16 Gyr ago have faded to a luminosity of $10^{-4.85} L_\odot$, or to approximately 0.9 magnitudes fainter than the disk cutoff. In this case, $x_f = 1$ out to where the HDF can see white dwarfs with $M_V = 17.1$ or brighter. From Table 1, if the limiting V magnitude of the HDF is 28.8, then $x_f = 1$ out to 2.2 kpc.

A second (more realistic) case allows for more rapid fading of white dwarfs at low luminosities (below $10^{-4.5}$ $L_\odot$) as the result of crystallization and Debye cooling. In this phase, a rough fit to the models of Winget et al. (1987), gives $t_{\rm cool} \approx 2.85 \times 10^8 L^{-1/3}$ yr; halo white dwarfs with an age of 16 Gyr have faded approximately 1.9 magnitudes below the disk cutoff. Assuming that star formation in the halo was continuous until the disk formed, 40% of halo white dwarfs have absolute magnitudes brighter than $M_V = 17.1$, compared with 100% in the previous limit. An upper limit to x_f in this case, with the same limiting

V and M_V as above, is $x_f = 0.4$ at 2.2 kpc. Of course, if star formation in the halo declines significantly between the initial collapse phases and the formation of the disk, the value of x_f would be further reduced. As an example, if halo star formation began 16 Gyr ago and lasted 2 Gyr, then the most luminous halo white dwarf would have $M_V \approx 17.6$, and (with a V magnitude limit on the HDF of 28.8) x_f would be zero beyond 1.7 kpc ($m_l - M_f > 11.2$). A younger halo age increases x_f for a given $m_l - M_f$; if the halo is only 12 Gyr old and star formation continued for 2 Gyr, the most luminous halo white dwarfs would then have $L = 10^{-4.64} L_\odot$, and x_f would be 0.94 under the conditons above ($m_l = 28.8, M_f = 17.1$).

Other more extreme cases can be considered, such as rapid and very intense bursts of star formation early in the history of the halo with a subsequent exponential decline in the birth rate with a time scale of less than 1 Gyr (Tamanaha et al. 1990). In such models, the luminosity function of halo white dwarfs can increase rapidly at luminosities below the disk cutoff, and still be consistent with the observations of Liebert et al. (1988). Parameters of these burst models include the age of the galaxy; if for example the age of the galaxy is significantly larger than 14 Gyr, then x_f would be correspondingly small; the earlier the burst, the smaller x_f is.

In all of the above cases, the value of x_f depends on the age of the halo; if the halo is 16 Gyr or older, then halo white dwarfs would have had sufficient time to fade below detectability on the HDF. However, since some current cosmological investigations favor an age for the Universe that is comparatively young (i.e. Freedman, et al. 1994), the possibility remains that a significant number of halo white dwarf stars population can be detected on the HDF if indeed they make up a significant fraction of the baryonic component of the dark halo of our Galaxy. Assuming then that there are a significant fraction of white dwarfs with $M_{\rm bol} < 18$, and considering the 3σ limiting magnitude of 30 in the red bands of the HDF, a total white dwarf population of $12x_f$ objects might exist in the HDF.

One can now ask if such halo white dwarfs should have been seen in studies of the white dwarf luminosity function that were restricted to the solar neighborhood. This issue has been addressed by Adams & Laughlin (1996) who show that the luminosity function of white dwarfs in the solar neighborhood as reported by Liebert et al. (1988) places strict upper limits on the numbers of halo white dwarfs at luminosities above the disk cutoff luminosity of $10^{-4.5} L_\odot$. The last data points in the disk white dwarf luminosity function are upper limits of approximately $5 \times 10^{-5} {\rm pc}^{-3} M_{\rm bol}^{-1}$ at this luminosity. For luminosity functions that derive from standard models of cooling white dwarfs, Adams & Laughlin (1996) show that their luminosity function must quickly rise to several $\times 10^{-4} {\rm pc}^{-3} M_{\rm bol}^{-1}$ and remain high down to very low luminosities if the age of this population is not excessively larger than the age of the oldest globular clusters.

Is the space density of the halo white dwarfs sufficiently low that it is not in conflict with the observed cutoff for disk stars? Assuming that the dark halo density is tracked by halo white dwarfs seen by MACHO, then the local space density of visible halo white dwarfs (that is, at the solar distance from the galactic center) should be given by $0.5\rho_o f(8.5)\mathrm{kpc})/<m_{\mathrm{wd}}>$. This is a space density of $8.3 \times 10^{-3}x_f$ halo white dwarfs per cubic parsec in the solar neighborhood. This places the nearest halo white dwarf about 5 pc away, on average.

4. Summary and Conclusions

There are no obvious stars in the HDF images (apart from a few "bright" 20th magnitude stars); the task of discriminating between stellar and nonstellar objects at very faint magnitudes requires extreme care. Still, if halo white dwarfs are old and therefore quite faint and red, there could indeed be halo white dwarfs hiding at slightly fainter magnitudes in the HDF – though not as many as suggested by the MACHO result (Adams & Lauglin 1996). A recent preprint by Flynn et al. (1996) reports that no stellar objects exist down to $V = 26.3$ for objects with $2.5 > V - I > 1.8$. For white dwarf stars, $V - I$ at the cool end of the white dwarf luminosity function is a the red end of this range. With a conservative value for the absolute magnitude of representative halo white dwarfs of $M_f = 18.0$ (i.e. ages of approximately 16 Gyr), this limit to V of 28.8 corresponds to $m_l - M_f$ of 10.8. Table 1 shows that approximately three halo white dwarfs should have been seen *if $x_f = 1$*. From the discussion of the previous section, x_f can range from 1 to nearly zero, depending on the population of halo stars that produced white dwarfs. A value of x_f of 0.4 is likely; thus Flynn et al. (1996) should have seen at least one or two white dwarfs. The fact that they saw none, though it fails to independently confirm the MACHO suggestion, is inconclusive. The dependence on x_f makes this a weak constraint, though models of star formation in the halo can easily produce more luminous white dwarfs than the $M_V = 18$ used above with nonzero values of x_f.

If the MACHO suggestion is correct, then a significant number of white dwarfs (3 times as many) could be detected if one could reach a magnitude limit for stellar objects that is fainter by one more magnitude, to $I = 27.3$, or $V = 29.8$ on the V exposures. Given the signal–to–noise statistics associated with the HDF exposures, along with judicious morphology and color cuts, such limits should be approachable. If the HDF still fails to yield white dwarf stars, then either 1) the MACHO suggestion that 50% of the mass of the baryonic halo is in the form of white dwarfs is incorrect, or 2) the value of x_f is significantly smaller than predicted with conventional models of white dwarf formation and evolution in

the halo. Already, observations of the disk white dwarf luminosity function require that x_f be small enough to allow a large enough number of white dwarfs in the solar neighborhood, as follows from the work of Adams & Laughlin (1996). Point 2 would in turn indicate that acceptance of the MACHO hypothesis implies a large and fast burst of star formation in the early years of the evolution of our Galaxy in excess of 16 Gyr ago, or that white dwarfs cool more quickly than conventional theoretical models predict below $M_{\rm bol} \approx 18.0$.

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Table 1: Number of halo white dwarfs expected on HDF.

$m_l - M_f$	d_f [kpc]	M_H [M$_\odot$]	N_{wd}/x_f
9.50	0.794	0.536	0.446
10.0	1.000	1.054	0.878
10.5	1.260	2.065	1.720
11.0	1.585	4.022	3.352
11.5	1.995	7.780	6.483
12.0	2.512	14.91	12.42
12.5	3.162	28.21	23.51
13.0	3.981	52.54	43.78
13.5	5.012	95.93	79.95